# Control Design and Simulation VIs and Functions

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**Installed With:** Control Design and Simulation Module. This topic might not match its corresponding palette in LabVIEW depending on your operating system, licensed product(s), and target.

Use the Control Design and Simulation VIs and Functions to design, analyze, simulate, and deploy dynamic system models.



**Note** The <u>System Identification</u> VIs enable you to create mathematical models of dynamic systems by using system identification methods.

Subpalette	Description
<u>Control Design</u> <u>VIs and</u> <u>Functions</u>	Use the Control Design VIs and functions to construct, analyze, and deploy dynamic system models in LabVIEW.
Simulation VIs and Functions	Use the Simulation VIs and functions to create simulation applications in LabVIEW.

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# Error Codes (Control Design and Simulation Module)

### **Control Design Error Codes**

The <u>Control Design</u> VIs can return the following error codes. Refer to the <u>KnowledgeBase</u> for more information about correcting errors in LabVIEW.

Code	Description
-41706	The number of inputs and outputs of the first model do not match the number of input and outputs of the second model.
-41705	The parallel interconnection with a transfer function model must have the same transport delay.
-41704	The number of inputs of the first model is not equal to the number of outputs of the second model. The number of inputs (columns) of Model 1 is not equal to the number of outputs (rows) of Model 2.
-41703	The denominator cannot equal zero. The denominator of the transfer function cannot equal zero.
-41702	At least one delay is less than zero.
-41701	The denominator must have one element. You did not specify the denominator in the transfer function. There must be at least one element in the denominator.
-41700	The numerator must have one element. You did not specify the numerator in the transfer function. There must be at least one element in the numerator.
-41699	Matrix R not provided. Matrix R not provided.
-41698	The dimension of w is not consistent with the dimensions of the stochastic state-space model. The dimension of w is not consistent with the dimensions of the stochastic state-space model.
-41697	The dimension of v is not consistent with the dimensions of the stochastic state-space model. The dimension of v is not consistent with the dimensions of the stochastic state-space model.
-41695	The cross-covariance matrix is not valid. The cross-covariance matrix is not valid. The compound auto-covariance and cross-covariance matrices must be positive semi-definite.

-41693	The dimension of E{w} is not proper. The dimension of E{w} is not proper. The dimension of E{v} must equal the number of states.
-41692	The dimensions of the covariance matrix are improper. The dimensions of the covariance matrix are improper.
-41691	The covariance matrix is not positive semi-definite. The covariance matrix is not positive semi-definite.
-41690	N is not valid. N is not valid. The matrix [Q N; N' R] must be positive semi-definite.
-41687	The R matrix is not positive definite. The R matrix is not positive definite.
-41685	The Q matrix is not symmetric. The Q matrix is not symmetric.
-41684	The covariance matrix is not symmetric. The covariance matrix is not symmetric.
-41681	Gain rows not equal to number of outputs The number of rows of the gain do not equal the number of outputs of the system model.
-41680	Gain columns are not equal to number of inputs. The number of columns of the gain do not equal the number of inputs of the system.
-41679	The system model does not have an input.
-41678	The index specified in the input, output, or state vector is greater than the maximum system dimension.
-41677	Matrix D not provided. You did not provide the required system model matrix D.
-41676	Matrix C not provided. You did not provide the required system matrix C.
-41675	Matrix B not provided. You did not provide the required system matrix B.
-41674	Matrix A not provided. You did not provide the required system matrix A.
-41673	Different number of columns for matrices R and Q in the Lyapunov equation. The number of columns for matrices R and Q must be identical in the Lyapunov equation.

-41672	Different number of rows for matrices P and Q in the Lyapunov equation. The number of rows for matrices P and Q must be identical in the Lyapunov equation.
-41671	Matrix R is not square in the Lyapunov equation.
-41670	Matrix P is not square in the Lyapunov equation.
-41669	Ackermann valid for single-output only. Ackermann is valid for single-output system models only. For Observer Gain, C must have one row.
-41668	The system model is not single-output.
-41667	The system model is not single-input.
-41666	The number of rows in D is not equal to the number of outputs.
-41665	The number of columns in D is not equal to the number of inputs
-41664	The number of columns in the regulator gain K does not equal the number of states.
-41663	The number of row in the regulator gain K does not equal the number of inputs.
-41662	N rows not equal to Nw. The number of rows in N is not equal to the dimension of the noise vector w (Nw).
-41661	The number of columns in N is not equal to the number of outputs.
-41660	The dimensions of R are not equal to number of outputs in the system model.
-41659	The number of rows in N is not equal to number of outputs. The number of rows in N is the number of outputs when output weighting.
-41658	A is ill-conditioned. You cannot calculate its inverse.
-41657	The system model is marginally stable. Calculations require a stable system model.
-41656	The system model is not stable Calculations require a stable system model.
-41655	The system model is not controllable or observable. The pair [A B] or [A C] is not controllable or observable.
-41654	The number of rows in H is not equal to the number of outputs.

-41653	The number of rows in G does not equal the number of states in the system model.
-41652	G columns not equal to H columns. The number of columns in G and H must be equal.
-41651	The dimensions of Q is not equal to the dimension of the process noise. The dimensions of matrix Q must be square with dimensions identical to the dimension of the noise vector w.
-41650	The dimensions of Q do not equal the number of outputs. Matrix Q must be square with a dimension equal to the number of outputs.
-41649	Compound noise matrix is not positive semi-definite. The compound noise covariance matrix, [G O; H I]*[Q N; N' R]*[G O; H I], is not positive semi-definite.
-41637	The dimensions of Q are not equal to the dimensions of A, which also are the number of states. The dimensions of Q are not equal to the dimensions of A, which also are the number of states.
-41636	The number of columns in C is not equal to number of states.
-41635	Matrix A is not square. The matrix A must be square.
-41634	The system model is not controllable. The system model is not controllable so you cannot calculate the matrix transformation T.
-41633	The number of closed-loop poles does not equal the number of columns in matrix A.
-41632	Ackermann valid for single-input only Ackermann is valid for single-input system models only. For controller gain, B must have one column.
-41631	The number of rows in B is not equal to the dimensions of A, which also are the number of states. The number of rows in B is not equal to the dimensions of A, which also are the number of states.
-41630	Not a complex conjugate pair. Complex closed-loop poles must be in conjugate pairs.
-41629	Matrix Q not provided. You must specify the required matrix Q.

	positive semi-definite.
-41578	The number of final constraints does not match the number of constrained variables.
-41577	The number of initial constraints does not match the number of constrained variables.
-41576	The size of a weight factor vector does not match the size of the corresponding weight matrix.
-41575	At least one of the weight matrices in the cost function is not square.
-41574	The size of the input change weight matrix in the cost function does not match the number of inputs in the controller model.
-41573	The size of the input weight matrix in the cost function does not match the number of inputs in the controller model.
-41572	The size of the output weight matrix in the cost function does not match the number of outputs in the controller model.
-41571	The initial conditions used to initialize the model predictive controller do not match the dimensions of the model system matrices in the controller.
-41570	The input frequency vector must be greater than zero. The input frequency vector must be greater than zero.
-41569	The closed-loop transfer function cannot be calculated. The output Y is not a function of the input U when a feedback connection is implemented. Therefore, the closed-loop transfer function can not be calculated.
-41568	The initial condition vector does not match the number of outputs in the system model. The number of elements in the initial condition vector does not match the number of outputs in the system model.
-41567	The size of the time vector is too large. The given initial time (t0), final time (tf), or time step (dt) require the size of the time vector to be greater than the maximum allowable size.
-41566	The initial frequency is greater than the final frequency. The initial frequency must be less than the final frequency.

-41565	The initial gain must be less than the final gain. The initial gain you entered is greater than the final gain you entered. The initial gain must be less than the final gain.
-41564	dB drop has to be negative. For a bandwidth calculation, the db drop has to be a negative number.
-41563	The size of the frequencies vector and response vector is not equal.
-41562	The interpolation frequency does not lie within the range of the frequencies. The interpolation frequency does not lie within the range of frequencies specified by the frequencies vector.
-41561	The Gaussian White Noise matrix must have same rows as number of inputs to the system. The Gaussian White Noise matrix must have same rows as number of inputs to the system and be a positive semi-definite matrix.
-41560	The system model has infinite covariance due to direct feedthrough. The system model has direct feed through, which means the matrix D is not zero. Continuous system models with direct feedthrough have infinite covariance.
-41559	The state covariance matrix has negative eigenvalues. The covariance response is invalid because the state covariance matrix has negative eigenvalues.
-41558	Number of applied inputs does not match with number of inputs in system model. The number of inputs applied to the system model does not equal the number of inputs in the system model. Columns of matrices B and D in a state-space model, or columns in transfer function or zero-pole-gain arrays must be equal to number of applied inputs.
-41557	The number of initial states do not match the number of states of the system model. The number of initial states do not match the number of states (the dimensions of Matrix A) of the system model.
-41556	All waveforms must have the same dt and t0. All the input waveforms must have the same sampling time, dt, and initial time, t0.
-41555	The time step (dt) and sampling time of the discrete system model must be equal.

-41554	The time step (dt) must be less than the final time (tf).
-41553	The time Step (dt) must be greater than zero.
-41552	The initial time (t0) must be greater than or equal to zero.
-41551	The final time (tf) must be greater than the initial time (t0)
-41550	Input system model must be a single-input single-output (SISO) model.
-41528	The matrix exponential calculation overflowed.
-41527	The model has discrete poles at zero.
-41526	The model has a pole at 1 with multiplicity greater than 6.
-41525	The model has a negative real pole with multiplicity greater than 2.
-41524	Sampling time must be positive. The sampling time must be greater than zero.
-41523	There is a repeated connection between interconnected models.
-41522	The system model must be proper to perform this function.
-41521	The system model has a delay. This VI does not support system models with delays.
-41520	The system model has a transport delay. This VI does not support system models with transport delays.
-41519	The system model has an output delay. This VI does not support system models with output delays.
-41518	The system model has an input delay. This VI does not support system models with input delays.
-41517	Not a second order system model. The system model must be a second order system model.
-41516	The system model is not square. The number of inputs does not equal the number of outputs
-41515	All variable names must begin with alphabetical letters.
-41514	The sampling time for this transformation produces an ill- conditioned system model.
-41513	The frequency must be greater than zero.
_/1510	The order of the polynomial must be larger than zero.

-41511	The system model must be continuous. To use this VI, the sampling time of the system model must equal to zero.
-41510	The system model must be discrete. To use this VI, the sampling time of the system model must not equal zero.
-41509	The dimension of output delay vector does not equal the number of outputs of the system model.
-41508	The dimension of the input delay does not equal the number of inputs of the system model.
-41507	The dimensions of the input/output delay matrices must equal the number of inputs and outputs of the system model.
-41506	The delay in the discrete system model must be an integer. The delay in discrete system model must be a integer multiple of the sampling time.
-41505	The number of inputs or outputs exceeds the total inputs or outputs of system model.
-41504	The number of outputs of the existing system model does not equal the number of outputs of the supplied system model. The number of outputs of the existing system model does not equal the number of outputs of the supplied system model. Dimensions of matrices C and D of each system model must be compatible.
-41503	The number of inputs of the existing system model does not equal the number of inputs of the new system model. The number of inputs of the existing system model does not equal the number of inputs of the new system model. Dimensions of matrices B and D of each system model must be compatible.
-41502	The number of states of the existing system model does not equal the number of states of the supplied system model. The number of states of the existing system model does not equal the number of states of the supplied system model. Dimensions of the matrix A of each model must be compatible.
-41501	The system model is discrete. The input system model needs to be a continuous system so you can convert it into its discrete equivalent. However the input system model is already discrete.

	greater than or equal to zero, but the value you supplied is negative.
41500	This VI does not support system models with delays. The delay information was ignored.
41501	The system model has a transport delay. This VI does not support system models with transport delays. The transport delay was ignored.
41502	The system model has an input delay. This VI does not support system models with input delays. The input delay was ignored.
41503	The system model has an output delay. This VI does not support system models with output delays. The output delay was ignored.
41504	The delay information was ignored.
41505	The system model is not proper. The order of the numerator polynomial is greater than the order of the denominator polynomial.
41506	Fractional delays in the discretization process were ignored.
41507	Second connector ignored. The second connector is ignored as the second system model is undefined.
41508	The components of the transport delay matrix could not all be distributed. The residual transport delay matrix contains nonzero elements.
41509	The results might be inaccurate. Try a different method.
41510	The conversion of the stable continuous model resulted in an unstable discrete-equivalent model. The matching frequency must be less than pi/T for the stable continuous model to convert to a stable discrete-equivalent.
41511	The conversion of the stable discrete model resulted in an unstable continuous-equivalent model. The matching frequency must be less than pi/T for the stable discrete model to convert to a stable continuous-equivalent.
41550	Phase margin is infinite. The gain does not cross 0 dB, therefore phase margin is infinite.
41551	Gain margin is infinite. The phase does not cross -180 degrees,

	therefore the gain margin is infinite.
41552	Magnitude does not drop below given dB value. The bandwidth cannot be determined because the magnitude does not drop below the given dB value.
41553	The actual final time (tf) is different from the supplied value. The values of the time step (dt) and the initial time (t0) cause the actual value of final time (tf) to be different from the supplied value.
41554	The 2-norm is infinite since the system model is not stable
41555	The infinity norm is infinite because system model is marginally stable. The infinity norm is infinite because system is marginally stable . The continuous system model has poles on an imaginary axis, or the discrete system model has poles on the unit circle.
41556	Roots for large gain values were not plotted. The closed-loop roots for large gain values were not plotted on the graph.
41557	The final frequency was reduced to equal the Nyquist frequency of discrete system model.
41558	The given step time (dt) and vector size limitations caused a reduction in the final time from its ideal value.
41559	The time step (dt) is not ideal. The time step (dt) is not ideal because of the large final time needed to show the complete dynamics of response.
41560	Initial conditions were ignored. The outputs are linearly dependent. The matrix C of the system model is not full row rank.
41561	Initial conditions were ignored. Initial conditions were ignored because the system model is not strictly proper.
41562	The system model has infinite covariance due to direct feedthrough. The system model has direct feed through, which means the matrix D is not zero. Continuous system models with direct feedthrough have infinite covariance.
41630	The matrices Q and/or R are close to zero norm.
41631	The system model has no specified states.

41632	The system model has no specified inputs.
41633	The system model has no specified outputs.
41634	Measured outputs and known/manipulated inputs ignored. When in stand-alone configuration, the measured outputs, known inputs, and manipulated inputs are ignored.
41635	The user-defined threshold has been surpassed. The Control Design and Simulation Module could not place the poles in the requested location.
41729	Removed residue from the denominator. The denominator was changed to one, because numerator is zero.
41799	Invalid inputs or outputs were ignored in producing the plots. The inputs or outputs/states that exceeded the total number of input or outputs/states of the system model were ignored in producing the plots.

### **Simulation Error Codes**

The <u>Simulation</u> VIs and functions can return the following error codes. Refer to the <u>KnowledgeBase</u> for more information about correcting errors in LabVIEW.

Code	Description
-2376	The transport delay is configured in a nondeterministic manner. If determinism is required, consider choosing a finite value for either the final time of the simulation or for the maximum delay of the transport delay block.
-2374	The negative slew rate must be less than or equal to the positive slew rate.
-2373	The version of LabVIEW you installed for this embedded device supports only the 1 kHz timing source of the Simulation Loop. To achieve loop rates other than 1 kHz, you must specify an external timing source.
-2372	The index table for a lookup table (LUT) must be non-decreasing.
-2371	The discrete sample period of each discrete function must be an integer multiple of the overall discrete step size of the simulation.
-2370	A single-input single-output (SISO) state-space model requires a B matrix with only one column and a C matrix with only one row.
-2369	The model you specified requires direct feedthrough. Open the configuration dialog box of this function and set the Feedthrough parameter to Direct.
-2367	The External Model Interface DLL returned an error.
-2366	The External Model Interface Node returned an error.
-2365	The order of the linear time-invariant (LTI) model must remain the same from the previous iteration.
-2364	The dimension of the multiple-input multiple-output (MIMO) linear time-invariant (LTI) model must remain the same from the previous iteration.
-2363	A state-space model with indirect feedthrough requires an empty or zero D matrix.

-2362	The number of channels must match the number of inequality constraints.
-2361	Insufficient number of user defined reference points. Ensure that any user-defined reference points are equally spaced according to the Initial Time, Final Time, and Step Size subparameters of the Solver Parameters parameter.
-2360	To use the Discrete States Only ODE solver, the simulation diagram must not contain any continuous functions.
-2359	The discrete step size must be an integer multiple of the continuous step size. Set the discrete step size to an integer multiple of the continuous step size. If you are using Auto Discrete Time, ensure that all discrete functions on the simulation diagram have a sample period (s) that is an integer multiple of the continuous step size.
-2358	A discrete function cannot accept a continuous model.
-2357	A continuous function cannot accept a discrete model.
-2355	The value of the Decimation parameter for the Collector function must be greater than or equal to 1.
-2354	The number of elements in the input array does not equal the number of columns in the gain matrix.
-2353	You cannot change the maximum delay while the simulation is running.
-2352	The delay must be less than or equal to the specified maximum delay.
-2351	The specified parameter is a vector. Enter a vector value.
-2350	The specified parameter is a scalar. Enter a scalar value.
-2349	The parameter name is not in the specified parameter list.
-2348	The given State Derivatives parameter is incompatible with the specified subsystem.
-2347	The given Outputs parameter is incompatible with the specified subsystem.
-2346	The given Inputs parameter is incompatible with the specified subsystem.
-2345	The given States parameter is incompatible with the specified

	subsystem.
-2344	The given Outputs parameter is incompatible with the specified subsystem.
-2343	The given Inputs parameter is incompatible with the specified subsystem.
-2342	The given States parameter is incompatible with the specified subsystem.
-2341	The initial time of the simulation cannot be greater than or equal to the final time.
-2340	The linearizer detected an internal error.
-2339	The ODE solver detected an internal error.
-2338	The ODE solver detected an internal error.
-2337	You can linearize only simulation subsystems.
-2336	The simulation diagram returned NaN to the ODE solver.
-2335	The simulation diagram returned Inf to the ODE solver.
-2334	An overflow occurred in the ODE solver.
-2333	The step size must be between the minimum and maximum step size.
-2332	The minimum step size must be less than or equal to the maximum step size.
-2331	The absolute tolerance and relative tolerance cannot both be zero.
-2330	The discrete step size must be an integer multiple of the step size.
-2329	The simulation step size cannot be zero.
-2328	You can use the Linearize Subsystem dialog box only on simulation subsystems.
-2327	You can use the Linearize Subsystem dialog box only if you have created a VI under My Computer in the Project Explorer.
-2326	An internal error has occurred within the LabVIEW Control Design and Simulation Module. If the problem persists, contact National Instruments technical support.
-2325	The ODE solver did not converge at the minimum step size.

-2324	The ODE solver cannot meet the error tolerance using the minimum step size.		
-2323	The simulation step size must be greater than zero.		
-2322	You selected a feedthrough behavior that is inconsistent with the specified discrete integration method. Launch the configuration dialog box for this function and change the Feedthrough or Discrete Integrator parameter.		
	The dimensions of the arrays for the lookup table are inconsistent.		
-2318	The dimensions of the parameter vectors of this function do not match.		
-2317	You selected a feedthrough behavior that is inconsistent with the specified discrete integration method. Launch the configuration dialog box for this function and change the Feedthrough or Discrete Integrator parameter.		
-2316	The order of the numerator must be greater than the order of the denominator.		
-2315	You must match complex entries in the Zero-Pole-Gain function with complex conjugates.		
-2314	For a transfer function with indirect feedthrough behavior, the order of the numerator must be less than or equal to the order of the denominator.		
-2313	The size of the initial condition vector is incorrect.		
-2312	The size of the input vector is incorrect for the MIMO system.		
-2311	The order of the model must not change from the previous iteration of the Simulation Loop.		
-2310	The dimensions of matrices A, B, C, and D are not consistent with each other.		
-2309	The period for this function must be greater than zero.		
-2308	The duty cycle must be between 0% and 100%.		
-2306	The frequency for this function must be greater than zero.		
-2305	<b>05</b> The target time for the Chirp Signal function must be greater the the simulation initial time.		

-2304	The upper limit for the Saturation function must be greater than or equal to the lower limit.
-2303	The switch on point for the Relay function must be greater than or equal to the switch off point.
	The quantization interval for the Quantizer function must be greater than zero.
-2301	The Simulation Converter failed to properly convert an expression.

## **Expired or invalid license**

The license for the LabVIEW Control Design and Simulation Module has expired or is invalid. You can purchase this product by visiting the <u>National Instruments Web site</u>. If you have already purchased this product, select **Help**»**Activate LabVIEW Components** to activate this product.

### Front panel terminals are not allowed inside Case structures on simulation diagrams

You cannot place <u>front panel terminals</u> inside a <u>Case structure on a</u> <u>simulation diagram</u>.

To correct this error, move the front panel terminal outside of the Case structure.

# Mathematical Model Definitions (Control Design and Simulation Module)

The LabVIEW Control Design and Simulation Module provides tools to study the dynamics of systems described by linear time-invariant (LTI) continuous and discrete models. You can create <u>deterministic state-space</u>, <u>transfer function</u>, and <u>zero-pole-gain models</u>. You also can create <u>stochastic state-space models</u> and the <u>second-order statistics noise</u> <u>models</u>. You can use these forms to describe both single-input single-output (SISO) and multiple-input multiple-output (MIMO) systems.

Continuous transfer function and zero-pole-gain models use the s variable to define time, whereas discrete transfer function and zero-pole-gain models use the z variable to define time. Continuous state-space models use the t variable to define time, whereas discrete state-space models use the k variable to define time.

### **Deterministic State-Space Model**

Continuous  $\dot{\mathbf{x}}^{(t)} = A \mathbf{x}^{(t)} + B \mathbf{u}^{(t)}$   $\mathbf{y}^{(t)} = C \mathbf{x}^{(t)} + D \mathbf{u}^{(t)}$ Discrete  $\mathbf{x}(k+1) = A \mathbf{x}(k) + B \mathbf{u}(k)$  $\mathbf{y}(k) = C \mathbf{x}(k) + D \mathbf{u}(k)$ 

#### **Stochastic State-Space Model**

Continuous	$\begin{aligned} \boldsymbol{x}(t) &= \boldsymbol{A}  \boldsymbol{x}(t) + \boldsymbol{B}  \boldsymbol{u}(t) + \boldsymbol{G}  \boldsymbol{w}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}  \boldsymbol{x}(t) + \boldsymbol{D}  \boldsymbol{u}(t) + \boldsymbol{H}  \boldsymbol{w}(t) + \boldsymbol{v}(t) \end{aligned}$
Discrete	$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{G}\boldsymbol{w}(k)$
	$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{H}\mathbf{w}(k) + \mathbf{v}(k)$
Second-Order Statistics Noise Model	$\boldsymbol{Q} = E\{\boldsymbol{w} \cdot \boldsymbol{w}^{T}\} - E\{\boldsymbol{w}\} \cdot E^{T}\{\boldsymbol{w}\}$
	$\boldsymbol{R} = \boldsymbol{E}\{\boldsymbol{v} \cdot \boldsymbol{v}^{T}\} - \boldsymbol{E}\{\boldsymbol{v}\} \cdot \boldsymbol{E}^{T}\{\boldsymbol{v}\}$
	$\mathbf{N} = E\{\mathbf{w} \cdot \mathbf{v}^{T}\} - E\{\mathbf{w}\} \cdot E^{T}\{\mathbf{v}\}$

where *t* is continuous time.

k is the model sampling time multiplied by the discrete time step, where the discrete time step equals 0, 1, 2, ...

*x* is the model state vector.

*u* is the model input vector.

y is the model output vector.

w is the process noise vector.

 $\boldsymbol{v}$  is the measurement noise vector.

**A** is an  $n \times n$  state matrix of the given model.

**B** is an  $n \times m$  input matrix of the given model.

C is an  $r \times n$  output matrix of the given model.

**D** is an  $r \times m$  direct transmission matrix of the given model.

*n* is the number of model states.

*m* is the number of model inputs.

*r* is the number of model outputs.

**G** is a matrix relating **w** to the model states.

*H* is a matrix relating *w* to the model outputs.

**Q** is the auto-covariance matrix of **w**.

*R* is the auto-covariance matrix of *v*.

N is the cross-covariance matrix between w and v.

E{} denotes the expected value or the mean of the enclosed term(s).

#### **Transfer Function Model**

# $\begin{array}{l} \textbf{SISO} & \textbf{MIMO} \\ \textbf{Continuous} & \\ \textbf{H}(s) = \frac{b_0 + b_1 s + \ldots + b_{m-1} s^{m-1} + b_m s^m}{a_0 + a_1 s + \ldots + a_{n-1} s^{n-1} + a_n s^n} & v_i = \sum_{j=1}^n \textbf{H}_{ij}(s) u_j \\ \textbf{Discrete} & \\ \textbf{H}(z) = \frac{b_0 + b_1 z + \ldots + b_{m-1} z^{m-1} + b_m z^m}{a_0 + a_1 z + \ldots + a_{n-1} z^{n-1} + a_n z^n} & v_i = \sum_{j=1}^n \textbf{H}_{ij}(z) u_j \end{array}$

#### Zero-Pole-Gain Model

$$\begin{array}{ll} \text{SISO} & \text{MIMO} \\ \text{Continuous } H(s) &= \frac{k(s-Z_1)(s-Z_2)\dots(s-Z_m)}{(s-P_1)(s-P_2)\dots(s-P_n)} & y_i = \sum_{j=1}^n H_{ij}(s)u_j \\ \text{Discrete} & H(z) &= \frac{k(z-Z[1])(z-Z[2])\dots(z-P[n])}{(z-P[1])(z-P[2])\dots(z-P[n])} & y_i = \sum_{j=1}^n H_{ij}(z)u_j \end{array}$$

where s is the Laplace variable and continuous time

z is discrete time

m is the order of the numerator polynomial function

*n* is the order of the denominator polynomial function

 $b_{\rm m}$  are the coefficients of the numerator polynomial function

 $a_{\rm n}$  are the coefficients of the denominator polynomial function

 $Z_{\rm m}$  are the locations of the model zeros

 $P_{\rm n}$  are the locations of the model poles

k is the gain of the model

 $H_{ij}$  is the transfer function or zero-pole-gain equation at the *i*<sup>th</sup> input and *j*<sup>th</sup> output of a MIMO model